# Bursting and large-scale intermittency in turbulent convection with differential rotation

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The tilting mechanism, which generates differential rotation in two-dimensional turbulent convection, is shown to produce relaxation oscillations in the mean flow energy integral and bursts in the global fluctuation level, akin to Lotka-Volterra oscillations. The basic reason for such behavior is the unidirectional and conservative transfer of kinetic energy from the fluctuating motions to the mean component of the flows, and its dissipation at large scales. Results from numerical simulations further demonstrate the intimate relation between these low-frequency modulations and the large-scale intermittency of convective turbulence, as manifested by exponential tails in single-point probability distribution functions. Moreover, the spatio-temporal evolution of convective structures illustrates the mechanism triggering avalanche events in the transport process. The latter involves the overlap of delocalized mixing regions when the barrier to transport, produced by the mean component of the flow, transiently disappears.

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Bursting in the fluctuation level and relaxation oscillations in the kinetic energy of differential rotation have recently been observed in numerical simulations of a wide variety of convectively driven systems, both plasmas and ordinary fluids [1–5]. Moreover, the origin of large-scale intermittency frequently measured in turbulent flow remains an outstanding enigma [6–8]. In this paper, we present general arguments supported by numerical simulations revealing the nature and intimate relation between these phenomena for two-dimensional turbulent convection.

Strong magnetic fields in plasmas and fast solid body rotation of ordinary fluids tend to make their low frequency collective motions essentially two-dimensional [9,10]. As a consequence, the equation of motion can be reduced to the form of a vorticity equation

$$\left(\frac{\partial}{\partial t}+\hat{\mathbf{z}}\times\nabla\psi\cdot\nabla\right)\nabla_\perp^2\psi=\mathcal{L}_\psi(\psi,\theta), \tag{1a}$$

where for simplicity we have applied slab coordinates with $\hat{\mathbf{z}}$ in the direction of the magnetic field or the rotation axis, and $\nabla_\perp$ denotes the gradient operator in the perpendicular plane. The left-hand side describes vorticity advection with the velocity $\mathbf{v}_\perp=\hat{\mathbf{z}}\times\nabla\psi$, while the operator $\mathcal{L}_\psi$ contains model dependent effects such as dissipation and coupling to other fields through, e.g., buoyancy or electric currents. For magnetized plasmas the stream function $\psi(\mathbf{x},t)$ may be identified with the electrostatic potential.

A self-consistent transport problem is settled when the vorticity equation is coupled to the evolution of an advected thermal field $\theta(\mathbf{x},t)$ governed by

$$\left(\frac{\partial}{\partial t}+\hat{\mathbf{z}}\times\nabla\psi\cdot\nabla\right)\theta=\mathcal{L}_\theta(\psi,\theta), \tag{1b}$$

where again the term on the right-hand side describes effects different from two-dimensional advection. The operators $\mathcal{L}_\psi$ and $\mathcal{L}_\theta$ couple the two equations, and under the Boussinesq approximation the corresponding terms are linear in the fields $\psi$ and $\theta$. We will presently consider the paradigmatic case of two-dimensional thermal convection [10–12]

$$\mathcal{L}_\psi(\psi,\theta)=-PR\frac{\partial\theta}{\partial y}+P\nabla_\perp^4\psi,\quad \mathcal{L}_\theta(\psi,\theta)=-\frac{\partial\psi}{\partial y}+\nabla_\perp^2\theta, \tag{1c}$$

where $x$ denotes the radial direction, $y$ represents the periodic azimuthal direction, and $\theta$ is the temperature deviation from the hydrostatic equilibrium. We have introduced the Prandtl and Rayleigh numbers $P$ and $R$, and spatial and temporal scales are normalized to the fluid layer depth and the associated thermal diffusion time, respectively. The same model equations pertain to electrostatic flute modes in nonuniformly magnetized plasmas.

An important property of many convection systems is that they are confined to geometries with spatial periodicity. There is thus the freedom for differential fluid rotation in the corresponding azimuthal directions. Generally, such symmetric flows are not driven by linear instability mechanisms but are subject to collisional damping. They can however be sustained by linearly unstable fluctuating motions through a tilting mechanism [10–12]. It is convenient to define the radial profile of any field, denoted by a zero subscript, as its spatial average over the periodic directions. In particular we introduce the azimuthally mean flow component

$$v_0(x,t)=\frac{1}{L}\int_0^L dy\,\frac{\partial\psi}{\partial x},$$

where $L$ is the periodicity length. Similarly, we define the spatial fluctuation of any field as the deviation from its profile, and denote this by an overtilde. Note that the mean flow $v_0$ is intrinsically incapable of mediating convective transport along the driving thermal gradients, and hence form a benign path for fluctuation energy.

Averaging Eq. (1a) over the periodic direction we obtain the generic equation for the mean flow component

$$\partial v_0/\partial t+\partial/\partial x\,(\widetilde{v}_x\widetilde{v}_y)_0=P\,(\partial^2 v_0/\partial x^2)\,.$$

 

While the term on the right-hand side describes viscous diffusion, the last term on the left-hand side shows the possibility of local flow generation by Reynolds stress. Further integrating over the radial domain, it is evident that this mechanism does not create net angular momentum. Hence the ensuing mean flows are intrinsically sheared corresponding to differential rotation.

Based on these fundamental properties, it is natural to separate the kinetic energy into two components comprised by the fluctuating and mean motions, defined respectively by

$$K(t)=\int d\mathbf{x}\frac{1}{2}(\nabla_{\perp}\tilde{\psi})^2, \quad U(t)=\int d\mathbf{x}\frac{1}{2}v_0^2,$$

where the integrals over $d\mathbf{x}$ extend over the whole fluid volume under consideration. The evolution of these energy integrals are readily derived from the vorticity equation (1a),

$$\frac{dK}{dt}=-\int d\mathbf{x}\,\tilde{\psi}\mathcal{L}_{\psi}(\psi,\theta)+\int d\mathbf{x}\,v_0\frac{\partial}{\partial x}(\tilde{v}_x\tilde{v}_y), \tag{2a}$$

$$\frac{dU}{dt}=-\int d\mathbf{x}\,\psi_0\mathcal{L}_{\psi}(\psi,\theta)-\int d\mathbf{x}\,v_0\frac{\partial}{\partial x}(\tilde{v}_x\tilde{v}_y). \tag{2b}$$

The first term on the right-hand side of the two equations contain respectively the linear instability drive and collisional dissipation of the sheared mean flows. The last term on the right-hand side of either equation shows the conservative transfer of kinetic energy between these linearly forced and damped modes. Upon integration by parts, the transfer term in Eq. (2a) may be written as

$$\int d\mathbf{x}\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}\frac{\partial v_0}{\partial x}=-\int d\mathbf{x}\left(\frac{\partial y}{\partial x}\right)_{\psi}\left(\frac{\partial\psi}{\partial y}\right)^2\frac{\partial v_0}{\partial x}.$$

The first expression above shows that convection cells tilted such as to transport positive azimuthal momentum up the gradient of a sheared mean flow will sustain the latter against collisional dissipation. Alternatively, the second relation indicates that modes with lines of constant phase which slope with the flow shear will give their energy to the latter.

Let us further consider the linear evolution of a plane wave component $\Psi\exp(ik_xx+ik_yy)$ of the stream function whose vorticity is subject to passive differential advection by a uniformly sheared flow [13]. Given the constant shear rate $v_0'$, the stream function at subsequent times is given by

$$\frac{\Psi(k_x^2+k_y^2)}{(k_x-v_0'k_yt)^2+k_y^2}\exp[i(k_x-v_0'k_yt)x+ik_yy].$$

From the exponent we see that the radial wave number changes linearly in magnitude with time, corresponding to a continuous increase of the tilting of the plane wave with the mean flow. This shearing effect will effectively channel fluctuation kinetic energy to larger radial wave numbers, leading to enhanced fluctuation dissipation. A broadening in the $k_x$ direction of the energy spectrum is thus expected when the differential rotation builds up. This mechanism has been considered as the crucial one for the self-regulation of convection systems [4,9,15]. Note however that $1/t^2$ asymptotic amplitude decay, a consequence of the conservation of enstrophy, shows that sheared structures will invariably loose their kinetic energy after a possible transient growth. Hence structures which are not forced to be tilted against the flow shear give their energy to the mean flow.

Since individual azimuthal mode numbers are not directly affected by the presence of differential rotation, the kinetic energy transfer process can be described by decomposing each mode into an amplitude $|\hat{\psi}_{k_y}(x)|$ and a phase $\delta_{k_y}(x)$ whose radial dependence gives the tilting of the mode $k_y$. The contribution from a spectrum of such modes to the mean flow acceleration is given by

$$\frac{\partial}{\partial x}(\tilde{v}_x\tilde{v}_y)_0=-\sum_{k_y>0}4k_y\frac{\partial}{\partial x}\left(|\hat{\psi}_{k_y}|^2\frac{\partial\delta_{k_y}}{\partial x}\right),$$

again indicating that a radially inhomogeneous phase is necessary for mean flow modification by fluctuating motions. This gives the evolution of the mean flow energy integral

$$\frac{dU}{dt}=-\int d\mathbf{x}\,\psi_0\mathcal{L}_{\psi}-\int dx\sum_{k_y>0}4v_0'k_y|\hat{\psi}_{k_y}|^2\frac{\partial\delta_{k_y}}{\partial x}.$$

For structures tilted by a sheared flow we intuitively expect the phase angle $\delta_{k_y}$ to be proportional to the shear rate $v_0'$. Indeed, for the passively advected plane wave considered above $\partial\delta_{k_y}/\partial x=k_x-v_0'k_yt$ showing that the kinetic energy transfer terms are proportional to the square of the shear rate and the amplitude of the fluctuating motions. Moreover, the presence of a seed shear flow will result in a definite direction for a massive kinetic energy transfer from the linearly forced modes, the fluctuating motions, to the linearly damped modes, the sheared mean flows. In the case of well-confined mode structures, the kinetic energy integrals evolve according to the Lotka-Volterra equations

$$\frac{dK}{dt}=(\gamma-\alpha U)K, \quad \frac{dU}{dt}=-(\mu-\alpha K)U. \tag{3}$$

Here $\gamma$ corresponds to the linear growth of the fluctuation level in the absence of differential rotation, as described by the first term on the right-hand side of Eq. (2a). The term proportional to $\mu$ represents collisional damping of differential rotation energy while the parameter $\alpha$ measures the efficiency of mean flow generation by fluctuating motions. The quasilinear origin of this self-regulation process was recently emphasized in Ref. [14].

To further demonstrate the occurrence of dynamical regulation, we next consider the thermal convection model (1c) and begin by employing the method of modal truncation. Apart from providing significant physical insight, such low-dimensional approaches often capture the basic mechanisms that continue to operate in the strongly nonlinear regime [10–12]. A sound truncation of Eq. (1) is given by Ref. [11]

$$\psi=\hat{\Psi}_{11}\sin\pi x\sin k_yy+\hat{\Psi}_{01}\sin\pi x+\hat{\Psi}_{12}\sin 2\pi x\cos k_yy,$$

$$\theta=\hat{\Theta}_{11}\sin\pi x\cos k_yy+\hat{\Theta}_{02}\sin 2\pi x+\hat{\Theta}_{12}\sin 2\pi x\sin k_yy.$$

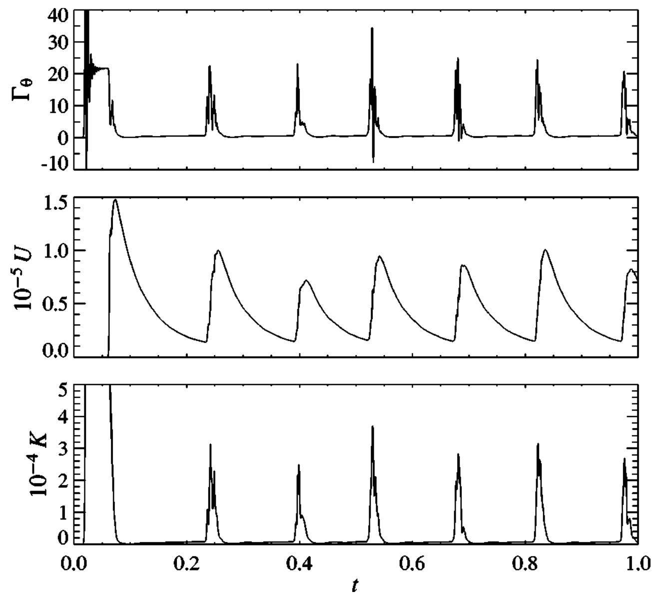


FIG. 1. Evolution of the domain integrated radial convective flux $\Gamma_\theta$, the mean flow kinetic energy $U$, and the fluctuation kinetic energy $K$.

Here and in the following we invoke boundary conditions corresponding to free-slip and stress-free, while the temperature is assumed to be constant on the radial boundaries. Apparently, the tilting of the convection cells in the above truncation is given by the presence of azimuthally phase-shifted higher radial harmonics (12) of the linearly driven modes (11). This also provides a path to fluctuation dissipation. In the limit of large $k_y$, the truncated evolution of the mode amplitudes may be reduced to four coupled ordinary differential equations, which may be written as [12]:

$$\dot{\Theta}_{02} = -\Theta_{02} - \Psi_{11}^2,$$

$$\dot{\Psi}_{11} = \gamma_{11}\Psi_{11} + \Psi_{11}\Theta_{02} - \Psi_{01}\Psi_{12},$$

$$\dot{\Psi}_{01} = -(P/4)\,\Psi_{01} + \{[3(1+P)]/4P\}\,\Psi_{11}\Psi_{12},$$

$$\dot{\Psi}_{12} = -\nu_{12}\Psi_{12} + \Psi_{11}\Psi_{01}.$$

A Lotka-Volterra model is now obtained by suppressing the temperature profile back-reaction $\Theta_{02}$ and slaving the linearly damped stream function mode $\Psi_{12}$ to $\Psi_{11}$ and $\Psi_{01}$. More generally, if we also slave the temperature mode $\Theta_{02}$, this reduces to the supercritical extension of the Lotka-Volterra model in which the additional nonlinear term $\lambda K^2$ appears in the evolution equation for the fluctuation energy. Similar equations were suggested by Diamond *et al.* [15] as a paradigmatic model for the transition to improved confinement regimes in magnetized plasmas. Again we emphasize the role of the conservative transfer of kinetic energy from the fluctuations to the mean flows involved in the reduction of radial convective transport, and do not allude to the turbulence shear decorrelation mechanism [4,9,15].

Finally, the fully nonlinear problem is addressed by means of numerical simulations of model (1). To this end we employ a hybrid finite difference–spectral code and present results for $R=10^6$ and unit Prandtl number and periodicity length. This example serves to demonstrate the general behavior observed for a wide range of parameters. The spatial resolution is 256 grid points in either direction and the initial condition is a periodic array of convection cells given by $\hat{\Psi}_{11}=10^{-5}$. In Fig. 1, we present the temporal evolution of the integrated radial convective heat flux $\Gamma = -\theta\,\partial\psi/\partial y$ and the kinetic energies. The mean flow energy displays relaxation oscillations, while the convective energy and transport show quasiperiodic bursts separated by quiet phases. Similar bursting is also observed for the temperature fluctuation level. This global behavior is readily understood in terms of the Lotka-Volterra equations (3). Initially the convective energy grows exponentially due to the primary instability. When the fluctuation level becomes sufficiently large to sustain the mean flows against collisional dissipation ($\alpha K > \mu$), this flow energy grows at the expense of the convective motions. The spatial fluctuations are effectively stabilized at a sufficiently large mean flow level ($\alpha U > \gamma$). Kinetic energy is however continuously transferred to the mean flows as far as $\alpha K > \mu$ leading to an almost complete suppression of the convective energy and thus the radial convective transport. Subsequently, there are no fluctuating motions to sustain the sheared flows which hence decay on a viscous time scale. Finally, as the mean flows become sufficiently weak ($\alpha U < \gamma$), the convective energy again starts to grow and the cycle repeats. This regulation results in a clear causality, manifested by a temporal phase-lag, between the two dynamics as observed in all the recent numerical studies [1–5].

The global bursting has profound influences on single-point recordings of the temperature and velocity fields as is

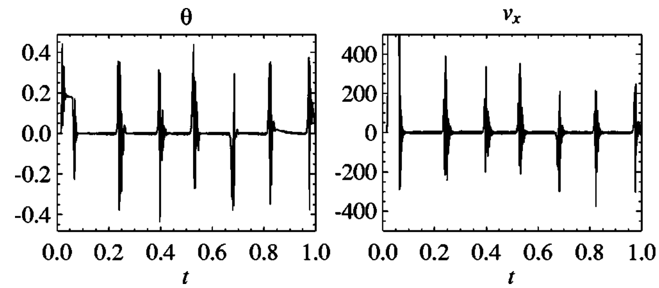


FIG. 2. Temperature to the left and radial velocity to the right measured in the center of the fluid layer, showing intermittent oscillations.

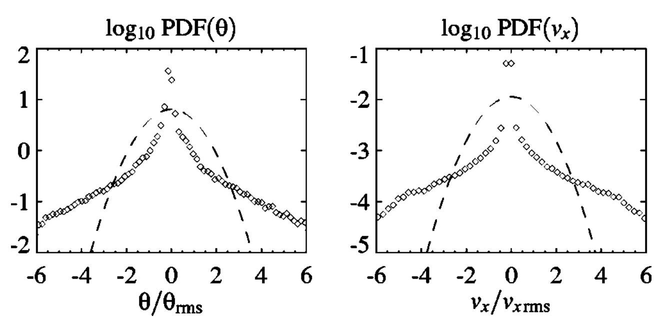


FIG. 3. Probability distribution function of temperature to the left and radial velocity to the right for the same signals as in Fig. 2. The broken lines show the fitted normal distributions.

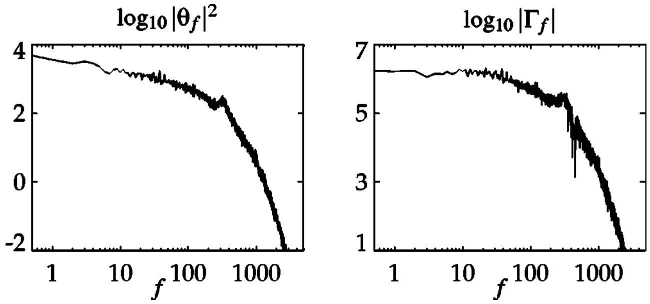


FIG. 4. Frequency power spectrum of temperature to the left and the flux spectral density to the right for the same signals as in Fig. 2.

readily seen in Fig. 2. The former gives its fingerprint on local probe measurements through a low-frequency modulation, resulting in the repetitive occurrence of large-amplitude events. Consequently, the associated probability distributions strongly deviate from normal statistics. Figure 3 indeed demonstrates the presence of large-scale intermittency manifested by exponential tails, a characteristic of self-organized turbulent convection [7,8]. The single-point recordings of the central temperature and radial velocity give flatness factors of 13 and 23, respectively, while there is no skewness due to the radial symmetry. The low-frequency modulations are also manifested in the spectral characteristics. This is observed as a significant energy content at small frequencies in the power spectra, presented in Fig. 4, which should not be confused with the violent dynamics during individual bursts.

Typical spatial fluctuation structures are presented in Fig. 5. In the quiet phases, the convection cells have small amplitudes and are localized at the radial boundaries where the mean flow shear vanishes. These convection cells form regions of closed streamlines where mixing occurs. They are separated by a zone of open streamlines in the azimuthal direction which effectively inhibits the radial convective transport. This transport barrier is produced by the mean component of the flow. As a consequence, when the mean flow is dissipated, the mixing regions overlap and merge in radially elongated convective cells, or streamers, while the fluctuation energy drastically rises. This results in a strong heat pulse and a radial transport avalanche. The local peak at large frequencies in the spectra seen in Fig. 4 may be associated with the chaotic transport during the periodic overlap of convection cells in their azimuthal propagation [2].

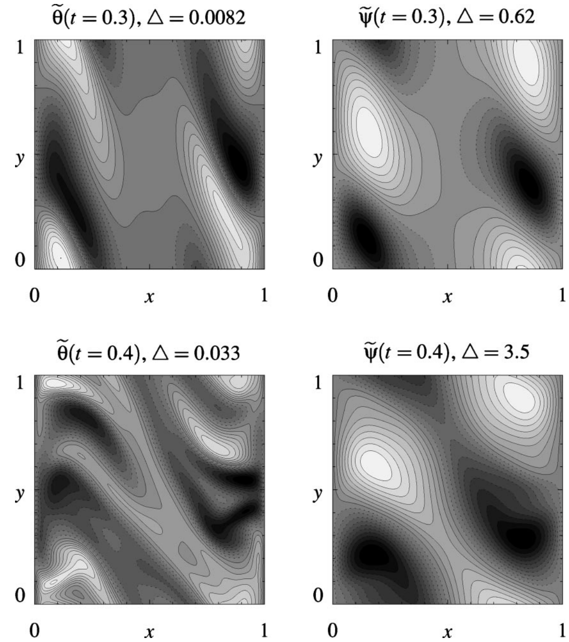


FIG. 5. Spatial fluctuation structure of the temperature and stream function in a quiet phase ($t = 0.3$) and during a burst ($t = 0.4$). The contour level spacing is given by the increment value $\triangle$.

In summary, we have within a general context discussed and elucidated the self-regulating nature of convection–shear flow systems. The unidirectional and conservative transfer of kinetic energy from the linearly forced fluctuating to the linearly damped mean flow results in global dynamics akin to Lotka-Volterra oscillations. Numerical simulations shows that this regulation is associated to the large-scale intermittency in strongly driven convective turbulence with differential rotation. Finally, the spatio-temporal evolution of convective structures illustrates the mechanism triggering transport avalanches.

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